Year 12 Mathematics Specialist 2018

Test Number 2: Functions and Graph Sketching

Resource Free

Name: SOLUTIONS **Teacher: DDA**

Marks: **45**

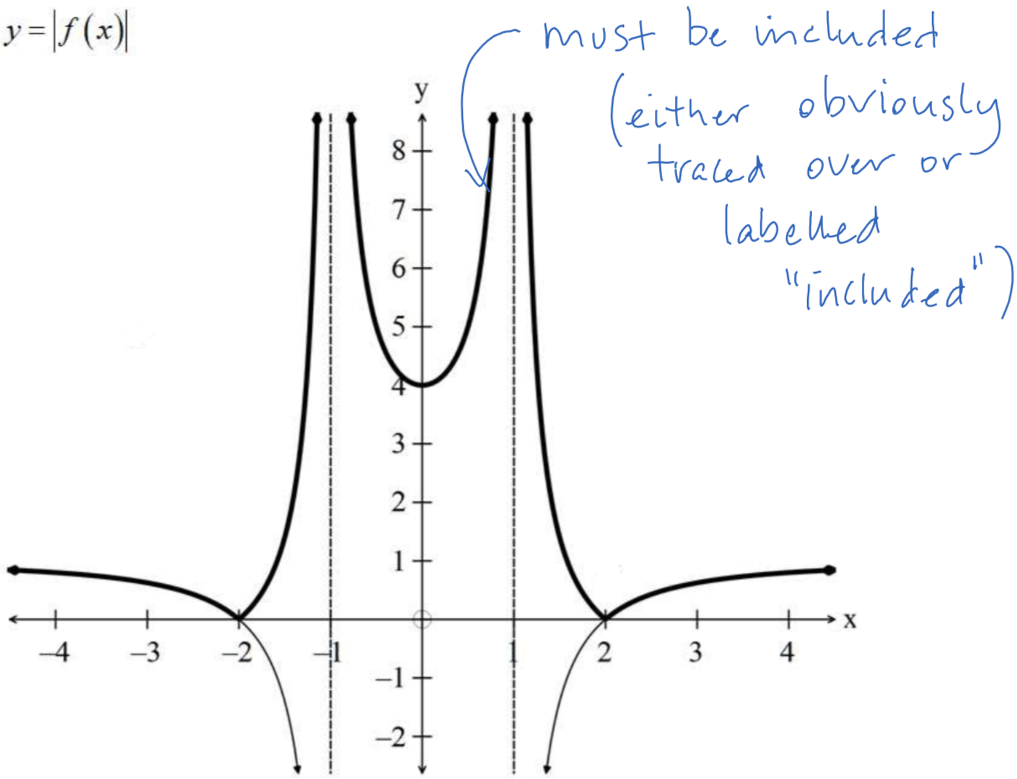
Time Allowed: **45 minutes**

Instructions: You **ARE NOT** permitted any notes or calculator. Show your working where appropriate remembering you must show working for questions worth more than 2 marks.

Question 1 **[5 marks]**

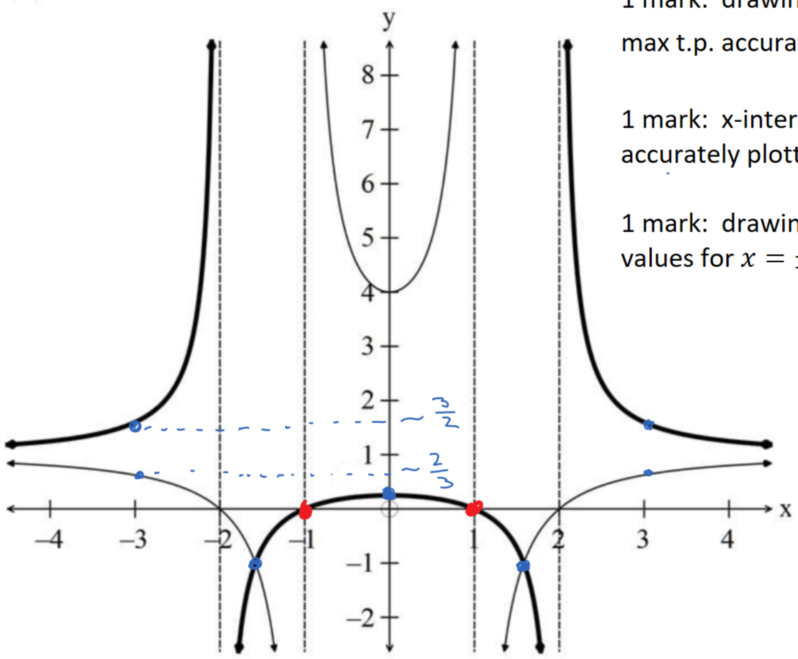
(a) Given the sketch of the function $f(x) = \dfrac{(x^2-4)}{(x^2-1)}$ sketch

(i) $y = |f(x)|$



(2)

(ii) $y = \dfrac{1}{f(x)}$



1 mark: drawing the concave down curve with max t.p. accurately plotted or labelled at $\left(0, \frac{1}{4}\right)$.

1 mark: x-intercepts at ± 1, and, points at $y = -1$ accurately plotted.

1 mark: drawing both 2 hyperbolic curves with y-values for $x = \pm 3$ being not above 3.

(3)

Question 2 **[10 marks]**

The function f is defined by $f(x)=\dfrac{x^2-6x+9}{x-2}$.

The first derivative of f is $f'(x)=\dfrac{x^2-4x+3}{(x-2)^2}$.

(a) State the coordinates of the y-axes intercept. (1 mark)

$$f(0)=\frac{9}{-2} \Rightarrow (0,\ -4.5)$$

(b) Determine the coordinates of the stationary points of the graph of $y=f(x)$. (3 marks)

$$x^2-4x+3=0 \Rightarrow (x-1)(x-3)=0$$

$$f(1)=-4,\ f(3)=0$$

$(1,-4)$ and $(3,0)$

(c) Determine the equations of all asymptotes of the graph of $y=f(x)$. (3 marks)

Vert asymptote: $x=2$

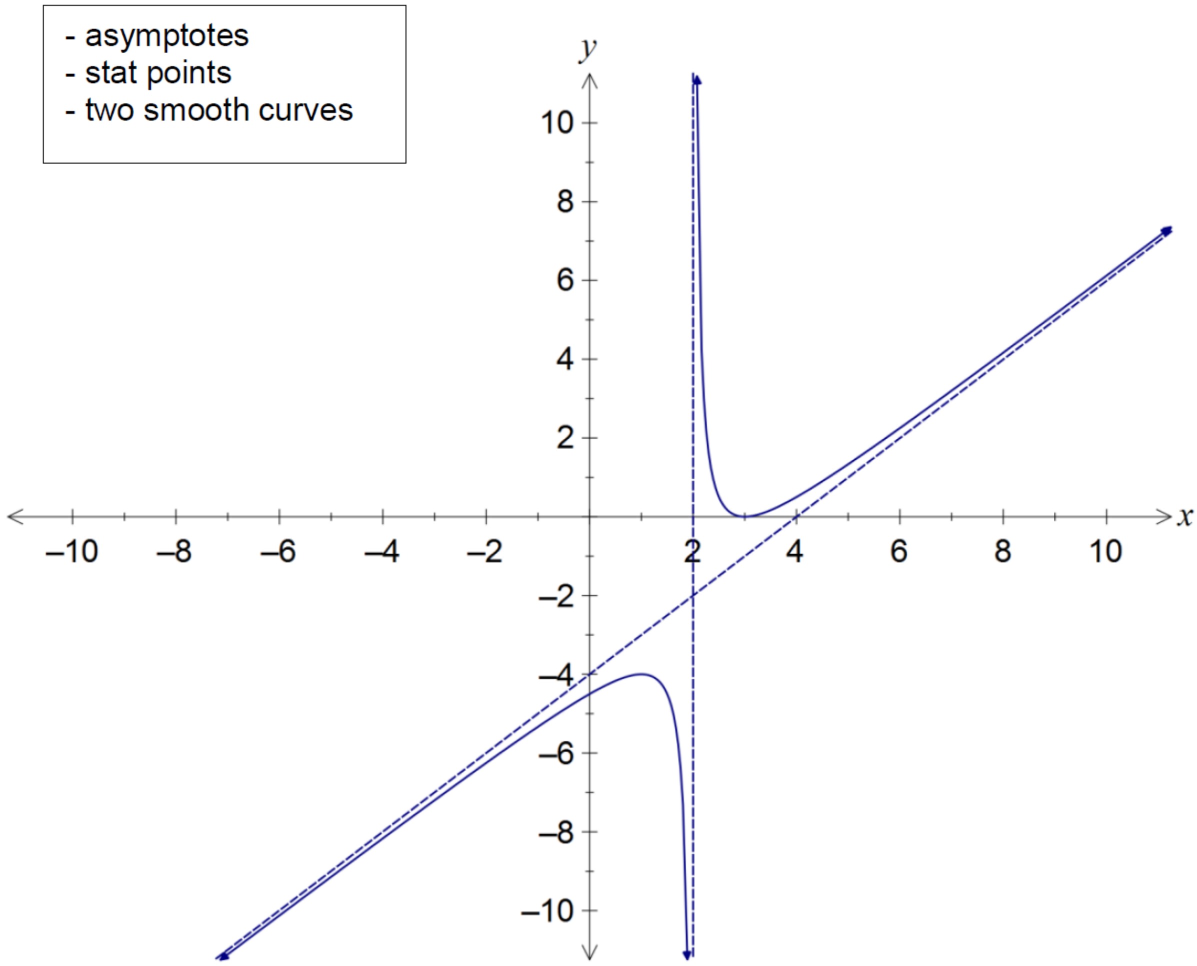
$$\frac{x^2-6x+9}{x-2}=x-4+\frac{1}{x-2}$$

Oblique asymptote: $y=x-4$

2	1	−6	9
	↓	2	−8
	1	−4	1

$x-4+\frac{1}{x-2}$

(d) Sketch the graph of $y = f(x)$ on the axes below. (3 marks)



Question 3 **[9 marks]**

Consider the function defined by $f(x) = \dfrac{1}{2x-1}$.

(a) State the natural domain for the function $f(x)$. (1 mark)

$$2x-1 \neq 0 \;\Rightarrow\; \left\{x : x \in \mathbb{R},\ x \neq \frac{1}{2}\right\}$$

(b) Determine the inverse of $f(x)$. (2 marks)

$$y = \frac{1}{2x-1}$$

$$2x-1 = \frac{1}{y}$$

$$x = \frac{1}{2y} + \frac{1}{2} = \frac{1+y}{2y}$$

$$f^{-1}(x) = \frac{1+x}{2x}$$

(c) Determine the composite function $f \circ f(x)$, expressing your answer as a single rational function. (3 marks)

$$f \circ f(x) = \frac{1}{2\left(\dfrac{1}{2x-1}\right)-1}$$

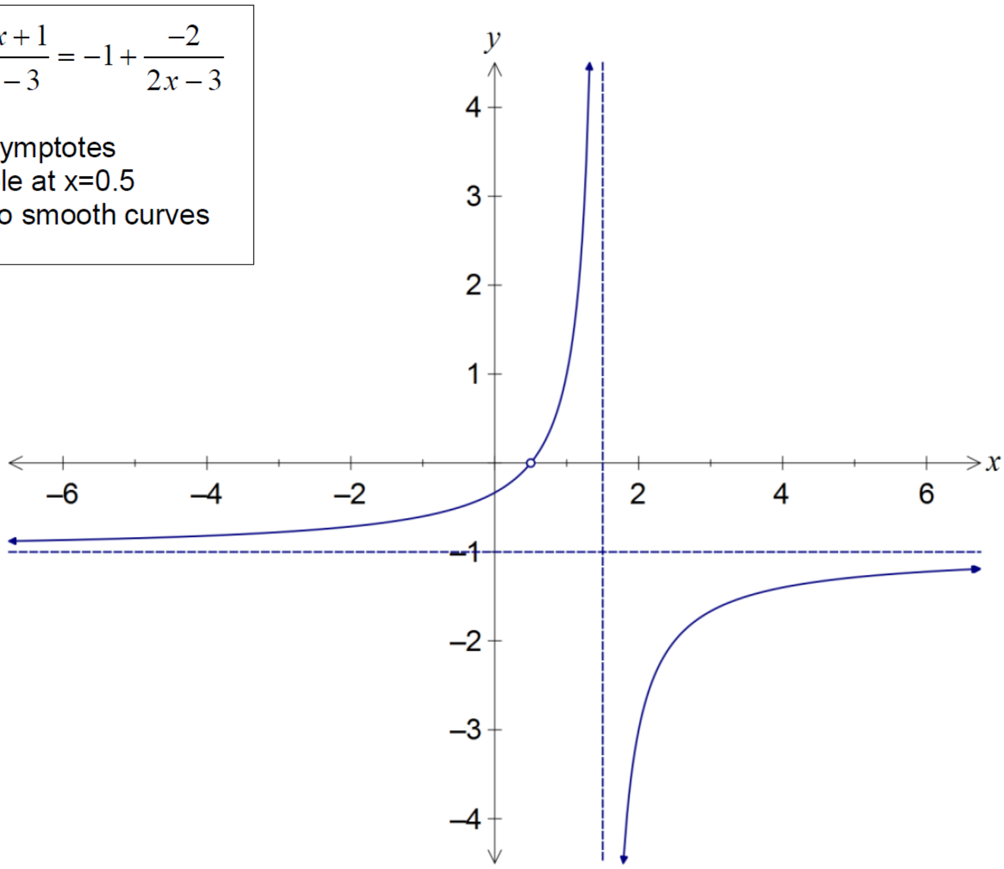
$$= 1 \div \frac{2-(2x-1)}{2x-1}$$

$$= \frac{-2x+1}{2x-3}$$

(d) Sketch the graph of $y = f \circ f(x)$ on the axes below. (3 marks)

$$\frac{-2x+1}{2x-3} = -1 + \frac{-2}{2x-3}$$

- asymptotes
- hole at x=0.5
- two smooth curves



hole

$\left\{x\epsilon\mathbb{R}: \boldsymbol{x} \neq \boldsymbol{\frac{1}{2}}\right\}$ $\overrightarrow{f(x)}$ $\{y \in \mathbb{R}: y \neq 0\}$

hole

$\left\{x\epsilon\mathbb{R}: x \neq \frac{1}{2}\right\}$ $\overrightarrow{f(x)}$ $\{y \in \mathbb{R}: \boldsymbol{y} \neq \boldsymbol{0}\}$

$\boldsymbol{x} \neq \frac{3}{2}$ ← $y \neq \frac{1}{2}$

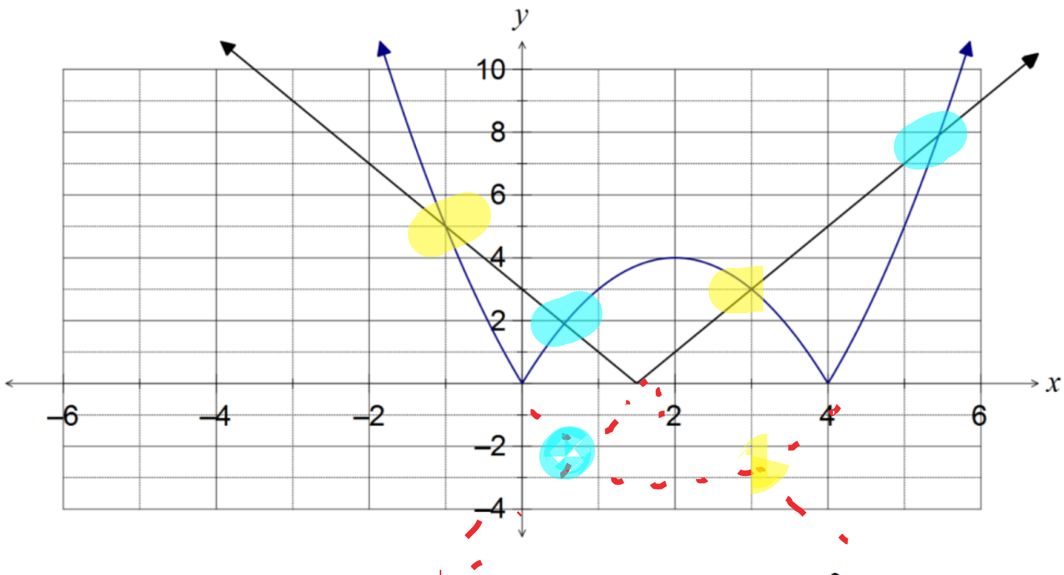
$x \neq 0$ → $\boldsymbol{y} \neq -\mathbf{1}$

asymptote

asymptote

Question 4 **[6 marks]**

The graph of $y = |f(x)|$ is shown, where $f(x) = 2x - 3$.



(a) Add the graph of $y = |g(x)|$ to the axes above, where $g(x) = (x-2)^2 - 4$. (2 marks)

(b) Solve $|f(x)| = |g(x)|$. (4 marks)

$(x-2)^2 - 4 = x^2 - 4x$

$x^2 - 4x = -2x + 3$

$x^2 - 2x - 3 = 0$

$(x+1)(x-3) = 0 \Rightarrow x = -1,\ 3$ (or from graph)

$x^2 - 4x = 2x - 3$

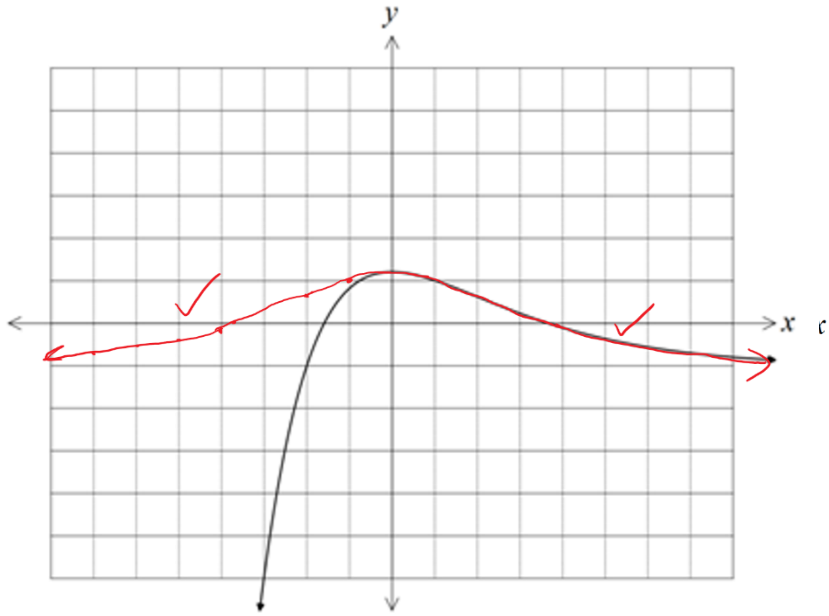
$x^2 - 6x + 3 = 0$

$(x-3)^2 = 6 \Rightarrow x = 3 \pm \sqrt{6}$

$x = -1,\ 3 - \sqrt{6},\ 3,\ 3 + \sqrt{6}$

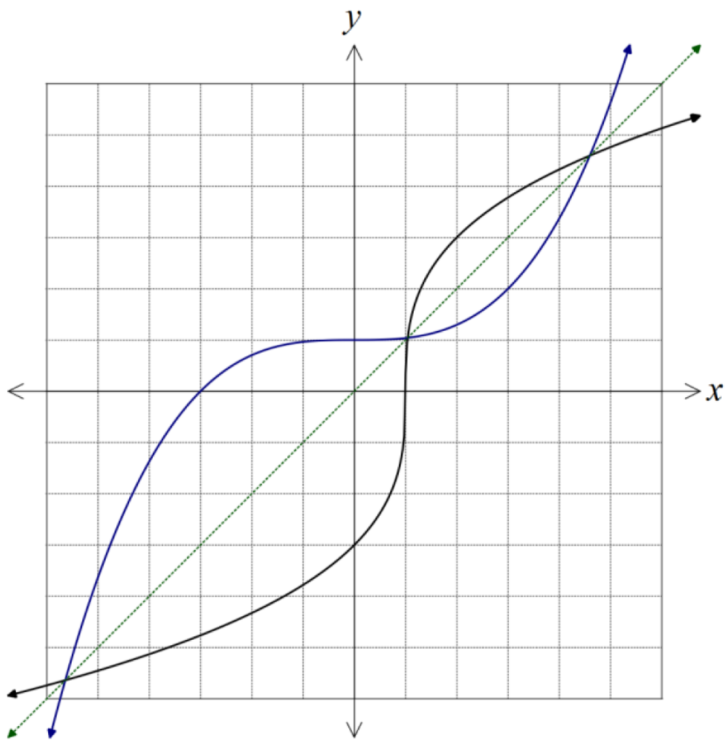
Question 5 **[4 marks]**

(a) The graph of $y = f(x)$ is shown below.
On the same axes, sketch the graph of $y = f(|x|)$ (2 marks)



(b) The graph of $y = h(x)$ is shown below.

On the same axes, sketch the graph of the inverse of h, $y = h^{-1}(x)$. (2 marks)

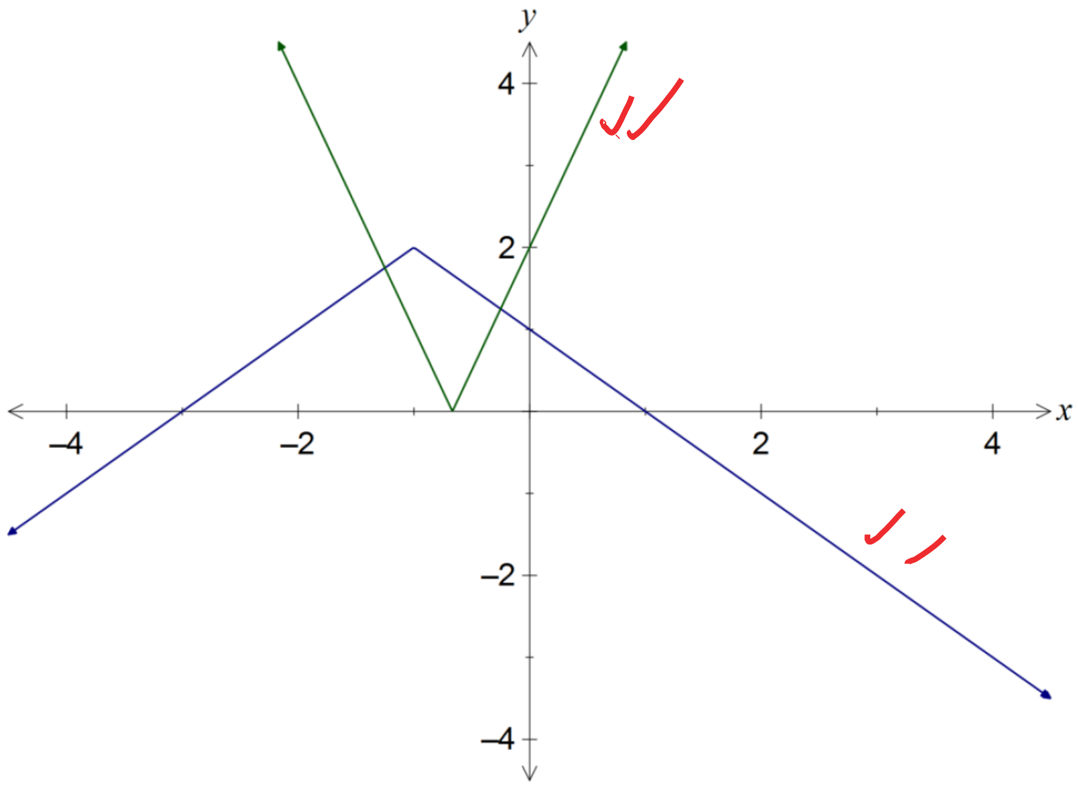


✓ accurately reflect across y=x

✓ y=x pts

Question 6 **[7 marks]**

On the axes below sketch the graphs of $y = 2 - |x+1|$ and $y = |3x+2|$, and hence solve the inequality $2 - |x+1| > |3x+2|$.



$x + 3 = -3x - 2 \Rightarrow x = -\frac{5}{4}$

$1 - x = 3x + 2 \Rightarrow x = -\frac{1}{4}$

Soln: $-\frac{5}{4} < x < -\frac{1}{4}$

Question 7 **[4 marks]**

For each of the following determine, with reasons, whether they are a 1-1 function, a many-to-one function or neither.

$$f(x) = x^3 - x, \qquad g(x) = \frac{1}{5} - x, \qquad x = y^2$$

$f'(x) = 2x^2 - 1 \qquad \therefore f'(x) > 0 \ \ if \quad 2x^2 > 1 \qquad but\ f'(x) \leq 0 \ \ is \ \ 2x^2 \leq 1$

Since $f(x)$ is neither constantly increasing nor decreasing it is many-to-one. ✓

$Let\ g(x_1) = g(x_2)$

$\frac{1}{5} - x_1 = \frac{1}{5} - x_2 \qquad \Rightarrow x_1 = x_2 \qquad \therefore g(x)$ is 1-1. ✓

$x = y^2 \qquad y = \pm 2 \quad \Rightarrow x = 4 \quad \therefore x = y^2$ is not a function at all. ✓

✓ at least 2 valid reasons